CORRELATIONS OF MASS-SHIFTED BOSONS

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We discuss a new kind of correlation, the correlation of particle-antiparticle pairs emitted back-to-back in the local rest frame of a medium, if medium effects cause mass modification of the quanta. The theory of these new correlations is formulated for relativistically expanding, locally thermalized sources.

1 Introduction

If hadronic mass-shifts in medium are accompanied with a sudden freeze-out, they were shown to result in squeezing of the fields [1,2] and as a consequence, new kind of back-to-back correlations of the observed quanta may appear [2,3]. The theoretical description of these back-to-back correlations were given first for static sources in refs. [2,3], however, the results were limited for those particles that are their own anti-particles, like the ϕ-s or π^0-s. The description of back-to-back correlations was generalized recently to particle - anti-particle pairs in refs. [4,5] and in ref. [6]. In the present contribution we discuss these new kind of back-to-back correlations following the lines of ref. [6]. We assume the validity of relativistic hydrodynamics up to freezeout. The local temperature field $T(x)$, the chemical potential distribution $\mu(x)$, the four-velocity distribution $u^\mu(x)$ and the oriented volume elements on freeze-out hypersurface $d^3\Sigma^\mu(x)$ are given on the freeze-out hypersurface Σ^μ, the space-time points are denoted by $x = (t, \mathbf{r})$. The temperature, flow and chemical potential distributions can be taken from realistic hydrodynamical calculations, for example, from refs. [7,8].

2 Model Assumptions:

The locally thermalized, relativistically expanding hydrodynamical fluid ensemble is specified by the density matrix

$$\hat{\rho} = \frac{1}{Z} \mathrm{e}^{-\frac{1}{(2\pi)^3}\int d^4k \int d^3\Sigma^\mu(x) k_\mu \, \hat{\mathcal{W}}(x,k)\, [k^\nu u_\nu(x) - \mu(x)]/T(x)}. \tag{1}$$

Here $\hat{\mathcal{W}}$ is the covariant Wigner operator[9], whose expectation value is the covariant phase space density, $\langle \hat{\mathcal{W}}(x,p)\rangle = 2\theta(p^0)\delta(p^2 - m^2) f(x,p)$, of on-shell

a-quanta in the semi-classical limit.

If we subdivide the fluid into locally thermalized, macroscopically infinitezimal fluid cells, the above hydrodynamic density matrix for locally thermalized systems can also be rewritten in the finite form

$$\rho = \prod_i \rho_i, \tag{2}$$

where the index i runs over all fluid cells. The locally thermalized density matrix in cell i is

$$\rho_i = \frac{1}{Z_i} \exp\left(-(H_i - \mu_i N_i)/T_i\right). \tag{3}$$

Consider, in the rest frame of each of the fluid elements, the following model Hamiltonian,

$$H_i = H_{0,i} - \frac{1}{2}\int d^3\mathbf{x} d^3\mathbf{y} \phi_i(\mathbf{x}) \delta M_i^2(\mathbf{x}-\mathbf{y}) \phi_i(\mathbf{y}), \tag{4}$$

where $H_{0,i}$ is the asymptotic Hamiltonian,

$$H_{0,i} = \frac{1}{2}\int d^3\mathbf{x} \left(\dot{\phi_i}^2 + |\nabla\phi_i|^2 + m_0^2\phi_i^2\right). \tag{5}$$

The scalar field $\phi_i(\mathbf{x})$ in this Hamiltonian, H_i, corresponds to quasi - particles that propagate in fluid element i with a momentum-dependent medium-modified effective mass, which is related to the vacuum mass, m_0, via

$$m_{*i}^2(|\mathbf{k}|) = m_0^2 - \delta M_i^2(|\mathbf{k}|). \tag{6}$$

The mass-shift is assumed to be limited to long wavelength collective modes in every fluid element:

$$\delta M_i^2(|\mathbf{k}|) \ll m_0^2 \qquad \text{if} \quad |\mathbf{k}| > \Lambda_s. \tag{7}$$

Here Λ_s stands for the scale above which the medium effects resulting in mass modifications become negligible.

We assume a sudden freeze-out. Mathematically this implies that before the break-up time H_i governs the time evolution of the fields in each fluid element, while after freeze-out, the quanta acquire their vacuum mass m_0 and the asymptotic Hamiltonian H_0 governs the time evolution. A more gradual freeze-out can be described by assuming a time-dependent mass-shift as in ref.[5]. In each cell, the post-freeze-out field can be expanded with creation and annihilation operators as

$$\phi_i(x) = \sum_{\mathbf{k}} (a_{i,\mathbf{k}} \varphi_{i,\mathbf{k}}(x) + h.c.), \tag{8}$$

a concept introduced to the field of Bose-Einstein correlation studies by Sinyukov and Makhlin in 1986-1988.

However, before the freeze-out the a quanta do not diagonalize the density matrix, due to the mass modification in the medium. It is possible to introduce new quanta in each fluid cell that diagonalize the density matrix of the medium (and *not* the Hamiltonian of the medium). These quanta are denoted by $b_{i,\mathbf{k}}$ and they are connected to the $a_{i,\mathbf{k}}$ quanta by a local two-mode Bogoljubov transformation as given in ref. [6].

Although the a quanta are observed, it is the b quanta that are thermalized in medium. The observable invariant single-particle and two-particle momentum distributions are given as:

$$N_1(\mathbf{k}_1) = \omega_{\mathbf{k}_1} \frac{d^3 N}{d\mathbf{k}_1} = \omega_{\mathbf{k}_1} \langle a^\dagger_{\mathbf{k}_1} a_{\mathbf{k}_1} \rangle, \tag{9}$$

$$N_2(\mathbf{k}_1, \mathbf{k}_2) = \omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2} \langle a^\dagger_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2} a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle, \tag{10}$$

$$\langle a^\dagger_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2} a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle = \langle a^\dagger_{\mathbf{k}_1} a_{\mathbf{k}_1} \rangle \langle a^\dagger_{\mathbf{k}_2} a_{\mathbf{k}_2} \rangle + \langle a^\dagger_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle \langle a^\dagger_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle + \langle a^\dagger_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2} \rangle \langle a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle, \tag{11}$$

where $a_{\mathbf{k}}$ is the annihilation operator for the asymptotic quantum with four-momentum $k^\mu = (\omega_{\mathbf{k}}, \mathbf{k})$, $\omega_{\mathbf{k}} = \sqrt{m^2 + \mathbf{k}^2}$ and the expectation value of an operator $\hat{O}$ is given by the medium-modified density matrix $\hat{\rho}_b$ as $\langle \hat{O} \rangle = \mathrm{Tr}\, \hat{\rho}_b\, \hat{O}$. Eq.(11) has been derived as a generalization of Wick's theorem for *locally* equilibriated (chaotic) systems in ref.[10]. The medium modified density matrix $\hat{\rho}_b$ has the same functional form as eq. (1), but for the medium-modified, mass-shifted b quanta. In order to simplify later notation, we introduce the chaotic and squeezed amplitudes [6] defined, respectively, as

$$G_c(1,2) = \sqrt{\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}} \langle a^\dagger_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle, \tag{12}$$

$$G_s(1,2) = \sqrt{\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle. \tag{13}$$

In most situations, the chaotic amplitude, $G_c(1,2) \equiv G(1,2)$ is dominant, and carries the Bose-Einstein correlations, while the squeezed amplitude, $G_s(1,2)$ vanishes. In this case, we recover from (11) the well-known two-particle inclusive correlation function given by

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1) N_1(\mathbf{k}_2)} = 1 + \frac{|G(1,2)|^2}{G(1,1)G(2,2)}, \tag{14}$$

which includes the effect of the two body correlations arising from the symmetrization of an ideal Bose gas.

For the hydrodynamic ensemble (1), eq.(14) reduces to the special form derived by Makhlin and Sinyukov[10]. In that case, the off-diagonal number

amplitude reads as

$$G(1,2) = \frac{1}{(2\pi)^3} \int d^3\Sigma_\mu K^\mu_{1,2} e^{iq^\nu_{1,2}x_\nu} \; f(x, K_{1,2}) \ . \tag{15}$$

The quantization relation, $[a_{\mathbf{k}_1}, a^\dagger_{\mathbf{k}_2}] = \delta^3(\mathbf{q}_{1,2})$, gives

$$\langle a_{\mathbf{k}_2} a^\dagger_{\mathbf{k}_1} \rangle \propto \int d^3\Sigma_\mu K^\mu_{1,2} e^{iq^\nu_{1,2}x_\nu} \; \left[f(x, K_{1,2}) + 1 \right]. \tag{16}$$

Note that the relative and average pair momentum coordinates, $\mathbf{q}_{1,2} = \mathbf{k}_1 - \mathbf{k}_2, \mathbf{K}_{1,2} = \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$, and $q^0_{1,2} = \omega_1 - \omega_2$ appear in (15,16). The validity of the approximations leading to (15,16) requires the width of $G(1,2)$ as a function of the relative momentum, $q = |\mathbf{q}_{1,2}|$, to be small. That width is given by $\sim 1/R$, where R is a characteristic dimension of the system. The semi-classical limit corresponds to $KR \gg 1$, where K is $|\mathbf{K}_{1,2}|$. Note that $\sqrt{\omega_{k_1}\omega_{k_2}} \sim K^0_{1,2}$ in this case. For $qR < 1$, the second term in (11) describes the minimal quantum interference associated with the indistinguishability of the bosons. The integration over the freeze-out three volume surface, $\Sigma^\mu(x)$, of the fluid is implemented with the invariant measure $d^3\Sigma_\mu K^\mu_{1,2}$ that reduces to $d^3\Sigma^\mu K_\mu = K^0 d^3x$ in the special case of a constant freeze-out time.

We can evaluate the chaotic and the squeezed amplitudes using the generalization of (15,16), noting that the b-quanta satisfy eq. (15) locally in each fluid cell,

$$\begin{aligned} G_c(1,2) &= \frac{1}{(2\pi)^3} \int d^3\Sigma_\mu K^{*\mu}_{1,2} e^{iq^{*\nu}_{1,2}x_\nu} |c_{1,2}|^2 n_{1,2} \\ &+ \frac{1}{(2\pi)^3} \int d^3\Sigma_\mu K^{*\mu}_{-1,-2} e^{iq^{*\nu}_{-1,-2}x_\nu} \; |s_{-1,-2}|^2 \left(n_{-1,-2} + 1 \right), \end{aligned} \tag{17}$$

$$\begin{aligned} G_s(1,2) &= \frac{1}{(2\pi)^3} \int d^3\Sigma_\mu K^{*\mu}_{1,-2} e^{iq^{*\nu}_{1,-2}x_\nu} \; c_{1,-2} s^*_{1,-2} n_{1,-2} \\ &+ \frac{1}{(2\pi)^3} \int d^3\Sigma_\mu K^{*\mu}_{2,-1} e^{iq^{*\nu}_{2,-1}x_\nu} \; c_{2,-1} s^*_{2,-1} \left(n_{2,-1} + 1 \right). \end{aligned} \tag{18}$$

The above expressions are invariant and defined even in the presence of a non-vanishing flow, as given below: The local squeezing parameter depends on a generalized energy ratio as

$$r(a,b,x) = \frac{1}{2} \log \left[\frac{K^\mu_{a,b} u_\mu(x) - \mu(x)}{K^{*\mu}_{a,b}(x) u_\mu(x) - \mu(x)} \right], \tag{19}$$

$$c_{a,b} = \cosh[r(a,b,x)], \quad s_{a,b} = \sinh[r(a,b,x)], \tag{20}$$

where $a, b = \pm 1, \pm 2$, the mean and the relative momenta for mass-shifted quanta are defined as $K^{*\mu}_{a,b}(x) = [k^{*\mu}_a(x) + k^{*\mu}_b(x)]/2$ and $q^*_{a,b} = k^{*\mu}_a(x) - k^{*\mu}_b(x)$.

In order to define the mass-shifted four-momenta and their back-to-back four-momenta in a covariant manner, we introduce

$$\tilde{k}^\mu(x) = k^\mu - u^\mu(x)[k^\nu u_\nu(x)], \tag{21}$$

$$\Omega_k(x) = u^\mu(x) k^*_\mu = \sqrt{m_0^2 + \tilde{k}^\mu \tilde{k}_\mu - \delta M_x^2(\tilde{k}^\mu \tilde{k}_\mu)}, \tag{22}$$

$$k^\mu_-(x) = 2u^\mu(x)[k^\nu u_\nu(x)] - k^\mu, \tag{23}$$

$$k^{*,\mu}(x) = u^\mu(x)\Omega_k(x) + \tilde{k}^\mu(x), \tag{24}$$

$$k^{*,\mu}_-(x) = u^\mu(x)\Omega_k(x) - \tilde{k}^\mu(x), \tag{25}$$

and use the short-hand notation $k^\mu_{-1} \equiv k^\mu_{1,-}$, $k^{*\mu}_{-1} \equiv k^{*\mu}_{1,-}$.

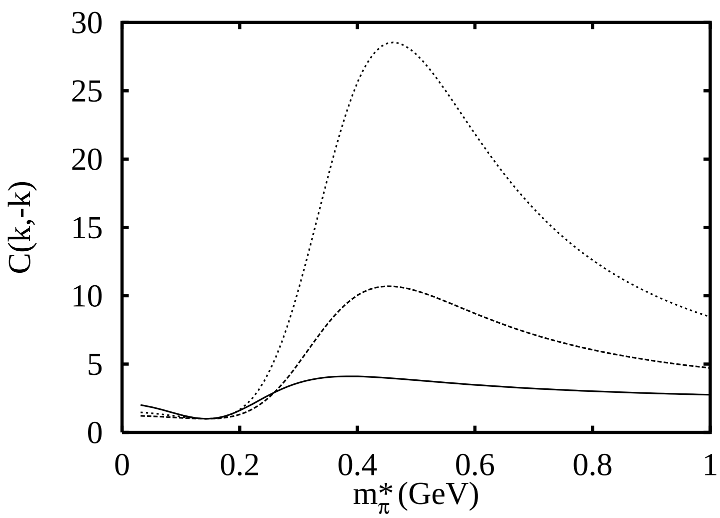

Fig. 1. Dependence of the strength of the back-to-back correlation function on the medium modified π meson mass, m^*_π, for π mesons at $|\mathbf{k}| = 0$, 300 and 500 MeV. This effect can be searched for both in (π^0, π^0) and in (π^+, π^-) correlation measurements at BNL or at CERN.

In eqs. (17,18) the Bose-Einstein distribution $n_{a,b}$ is

$$n_{a,b}(x) = \frac{1}{\exp\left[\frac{K^{*\mu}_{a,b}(x) u_\mu(x) - \mu(x)}{T(x)}\right] - 1}, \tag{26}$$

If the mass-shift is non-vanishing in a finite medium, as a consequence, a local squeezing is present (19), therefore the following new expressions are found for the particle spectra and the correlation function:

$$N_1(\mathbf{k}_1) = G_c(1,1), \tag{27}$$
$$N_2(\mathbf{k}_1,\mathbf{k}_2) = G_c(1,1)G_c(2,2) + |G_c(1,2)|^2 + |G_s(1,2)|^2, \tag{28}$$
$$C_2(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)}. \tag{29}$$

2.1 Particle-antiparticle correlations

As the Bogoliubov transformation mixes particles with anti-particles[4,11,12], the above considerations hold only for particles that are their own anti-particles, e.g. the ϕ meson and π^0. However, the extension to particle − anti-particle correlations is straightforward. Let + label particles, − antiparticles if antiparticle is different from particle, let 0 label both particle and antiparticle if they are identical particles. The non-trivial correlations from mass-modification for pairs of $(++)$, $(+-)$ and (00) type read as follows:

$$C_2^{++}(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)}, \tag{30}$$
$$C_2^{+-}(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)}, \tag{31}$$
$$C_2^{00}(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)}, \tag{32}$$

where we have assumed that mass-modifications of particles and anti-particles are the same as happens at vanishing baryon density.

Fig. 1 shows the strengh of the back-to-back correlation function for mass-shifted π mesons. The strengh of the back-to-back correlations is expected to increase with increasing momentum of one of the particles from the back-to-back particle pairs.

The essential properties of the HBT and the back-to-back correlations are illustrated on Figure 2. The HBT correlations appear at small relative momenta and the widths of these correlations measure the lengths of homogeneity, the size of the particle source within which the momentum space distribution of particles is similar, a concept introduced by Sinyukov and collaborators [10]. In contrast, back-to-back correlations measure the lengths of *in*homogeneity, the size of region where particle - anti-particle pairs with opposite momenta are created. The back-to-back correlations are limited to a region in momentum

space, where the medium mass-modifications are non-vanishing, as indicated by the cut-off Λ_s.

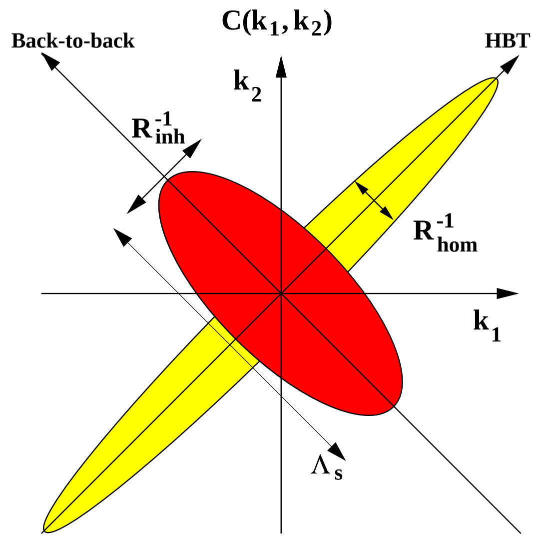

Fig. 2. Illustation for the essential properties of the back-to-back correlation function as created by a modification of hadronic masses in medium. The back-to-back correlations are present on the scale $\Lambda_s > |\mathbf{k}_1|, |\mathbf{k}_2|$, and their inverse width is measuring the lengths of *in*homogeneity, R_{inh}. In contrast, the inverse width of HBT correlations measure the lengths of homogeneity R_{hom}.

3 Highlights:

Back-to-back correlations of particle - anti-particle pairs were discussed for locally thermalized, medium modified bosons in a covariant framework. These theoretically predicted, new kind of correlations could be looked for at present and future relativistic heavy ion collisons at CERN SPS and LHC as well as at BNL AGS and RHIC experiments. The magnitude of the back-to-back correlations is expected to be surprisingly large. They are expected to carry information about the total longitudinal extension of inhomogenous, expanding sources. Our results will have implications also to the signal of DCC formation in two-pion correlations [5]: mass modifications of quanta are essential ingredients for a Disoriented Chiral Condensate (DCC) formation although

the thermal averaging performed here is not needed to describe DCC. Another interesting application of our results could be the study of particle production via the parametric resonance method in the context of reheating in the inflatory Universe [14]. We have solved the quantum optical problem of squeezing in a finite volume as well [13].

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